

Energy Spectrum for Neutral Collective Excitations in Striped Hall States

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Abstract. Neutral collective excitations in the striped Hall state are studied by using the single mode approximation and Hartree-Fock approximation at the half-filled third and fourth Landau level. We find that the spectrum includes anisotropic NG modes and periodic line nodes.

1. Introduction

In recent experiments, highly anisotropic states, which have anisotropic longitudinal resistances, were observed around the half-filled third and higher Landau levels[1, 2]. It is believed that the anisotropic state is the striped Hall state which is a unidirectional charge density wave in the mean field theory[3, 4]. The anisotropy of the resistance is naturally explained by the anisotropic Fermi surface in the magnetic Brillouin zone[5, 6, 7].

In the absence of disorder and edges, the quantum Hall system has the magnetic translation and rotation symmetry. In the striped state, a magnetic translation in one direction is spontaneously broken to the discrete translation and the rotation is also spontaneously broken to the π -rotation. Using the conserved current, the property of the neutral collective excitations is studied. The spectrum of the neutral collective excitation is obtained in the single mode approximation numerically[8]. The single mode approximation is useful for the fractional quantum Hall state (FQHS). We show that the spectrum for the striped Hall state has a multiple line node structure and cusps. Furthermore, the spectrum has anisotropic feature, that is, in one direction the spectrum resembles the liquid Helium spectrum with the phonon and roton minimum, and in another direction it resembles the FQHS spectrum.

2. Symmetries

Let us consider the two-dimensional electron system in a uniform magnetic field $B = \partial_x A_y - \partial_y A_x$. We ignore the spin degree of freedom and use the natural unit ($\hbar = c = 1$) in this paper. We introduce two sets of coordinates, relative coordinates and guiding center coordinates. The relative coordinates are defined by

$$\xi = \frac{1}{eB}(-i\partial_y + eA_y), \quad \eta = -\frac{1}{eB}(-i\partial_x + eA_x). \quad (1)$$

The guiding center coordinates are defined by

$$X = x - \xi, \quad Y = y - \eta. \quad (2)$$

These coordinates satisfy the following commutation relations,

$$\begin{aligned} [X, Y] &= -[\xi, \eta] = i/eB, \\ [X, \xi] &= [X, \eta] = [Y, \xi] = [Y, \eta] = 0. \end{aligned} \quad (3)$$

The operators X and Y are the generators of the magnetic translations of the one-particle state in $-y$ direction and x direction respectively. The angular momentum J is written as

$$J = \frac{eB}{2}(\xi^2 + \eta^2 - X^2 - Y^2). \quad (4)$$

J is the generator of the rotation of the one-particle state.

The total Hamiltonian H for the interacting charged particles is the sum of the free Hamiltonian H_0 and the Coulomb interaction Hamiltonian H_{int} as follows,

$$\begin{aligned} H &= H_0 + H_{\text{int}}, \\ H_0 &= \int d\mathbf{r} \Psi^\dagger(\mathbf{r}) \frac{m\omega_c^2}{2} (\xi^2 + \eta^2) \Psi^\dagger(\mathbf{r}), \\ H_{\text{int}} &= \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \Psi^\dagger(\mathbf{r}) \Psi^\dagger(\mathbf{r}') V(\mathbf{r} - \mathbf{r}') \Psi(\mathbf{r}') \Psi(\mathbf{r}), \end{aligned} \quad (5)$$

where Ψ is the electron field operator, $\omega_c = eB/m$ and $V(\mathbf{r}) = q^2/r$ ($q^2 = e^2/4\pi\epsilon$, ϵ is the dielectric constant) for the Coulomb potential.

Conserved charges are obtained as the spatial integral of the zeroth component of the Noether currents for the symmetries. We define the conserved charges, Q , Q_X , Q_Y , and Q_J for $U(1)$, magnetic translations, and rotation symmetry respectively, as follows,

$$\begin{aligned} Q &= \int j^0(\mathbf{r}) d\mathbf{r}, \\ Q_X &= \int j_X^0(\mathbf{r}) d\mathbf{r}, \quad Q_Y = \int j_Y^0(\mathbf{r}) d\mathbf{r}, \\ Q_J &= \int j_J^0(\mathbf{r}) d\mathbf{r}. \end{aligned} \quad (6)$$

Noether currents are defined by

$$\begin{aligned} j^\mu &= \text{Re}(\Psi^\dagger v^\mu \Psi), \\ j_X^\mu &= \text{Re}(\Psi^\dagger v^\mu X \Psi) - \frac{1}{eB} \delta_y^\mu \mathcal{L}, \\ j_Y^\mu &= \text{Re}(\Psi^\dagger v^\mu Y \Psi) + \frac{1}{eB} \delta_x^\mu \mathcal{L}, \\ j_J^\mu &= \text{Re}(\Psi^\dagger v^\mu J \Psi) + \epsilon_{0\mu i} x^i \mathcal{L}, \end{aligned} \quad (7)$$

where $v^\mu = (1, \mathbf{v})$, $\mathbf{v} = \omega_c(-\eta, \xi)$, Re means real part, and \mathcal{L} is the Lagrangian density for the total Hamiltonian H . These charges commute with the total Hamiltonian H and obey the following commutation relations,

$$\begin{aligned} [Q_X, Q_Y] &= \frac{i}{eB} Q, \\ [Q_J, Q_X] &= iQ_Y, \\ [Q_J, Q_Y] &= -iQ_X. \end{aligned} \quad (8)$$

The $U(1)$ charge Q commutes with all conserved charges. We assume that Q is not broken and the ground state is the eigenstate of Q as $Q|0\rangle = N_e|0\rangle$ where N_e is a number of electrons.

The commutation relations between the conserved charges and the current density operators read,

$$\begin{aligned}
[Q_X, j^\mu] &= -\frac{i}{eB} \partial_y j^\mu, \quad [Q_Y, j^\mu] = \frac{i}{eB} \partial_x j^\mu, \\
[Q_X, j_X^\mu] &= -\frac{i}{eB} \partial_y j_X^\mu, \quad [Q_Y, j_X^\mu] = \frac{i}{eB} \partial_x j_X^\mu - \frac{i}{eB} j^\mu, \\
[Q_X, j_Y^\mu] &= -\frac{i}{eB} \partial_y j_Y^\mu + \frac{i}{eB} j^\mu, \quad [Q_Y, j_Y^\mu] = \frac{i}{eB} \partial_x j_Y^\mu, \\
[Q_X, j_J^\mu] &= -\frac{i}{eB} \partial_y j_J^\mu - i j_Y^\mu, \\
[Q_Y, j_J^\mu] &= \frac{i}{eB} \partial_x j_J^\mu + i j_X^\mu \\
[Q_J, j^\mu] &= i(x\partial_y - y\partial_x)j^\mu + i\epsilon_{0\mu\nu}j^\nu.
\end{aligned} \tag{9}$$

The U(1) charge Q commutes with all Noether currents. If the expectation value of the right hand side of these equation is not zero, corresponding symmetry is spontaneously broken.

We use the von Neumann lattice (vNL) base for the one-particle states[9]. A discrete set of coherent states of guiding center coordinates,

$$\begin{aligned}
(X + iY)|\alpha_{mn}\rangle &= z_{mn}|\alpha_{mn}\rangle, \\
z_{mn} &= a(mr_s + i\frac{n}{r_s}), \quad m, n; \text{ integers,}
\end{aligned} \tag{10}$$

is a complete set of the (X, Y) space. These coherent states are localized at the position $a(mr_s, n/r_s)$, where a positive real number r_s is the asymmetric parameter of the unit cell and $a = \sqrt{2\pi/eB}$. By Fourier transforming these states, we obtain the orthonormal basis in the momentum representation

$$\begin{aligned}
|\beta_{\mathbf{p}}\rangle &= \sum_{mn} e^{ip_x m + ip_y n} |\alpha_{mn}\rangle / \beta(\mathbf{p}), \\
\beta(\mathbf{p}) &= (2\text{Im}\tau)^{1/4} e^{\frac{i\tau p_y^2}{4\pi}} \vartheta_1\left(\frac{p_x + \tau p_y}{2\pi}|\tau\right),
\end{aligned} \tag{11}$$

where ϑ_1 is a Jacobi's theta function and $\tau = ir_s^2$. The two-dimensional lattice momentum \mathbf{p} is defined in the Brillouin zone (BZ) $|p_i| < \pi$. Next we introduce a complete set in (ξ, η) space, that is the eigenstate of the one-particle free Hamiltonian,

$$\frac{m\omega_c^2}{2}(\xi^2 + \eta^2)|f_l\rangle = \omega_c(l + \frac{1}{2})|f_l\rangle. \tag{12}$$

The Hilbert space is spanned by the direct product of these eigenstates

$$|l, \mathbf{p}\rangle = |f_l\rangle \otimes |\beta_{\mathbf{p}}\rangle, \tag{13}$$

where l is the Landau level index and $l = 0, 1, 2, \dots$. We set $a = 1$ ($eB = 2\pi$) in the following calculation for simplicity.

Electron field operator is expanded by the vNL base as

$$\Psi(\mathbf{r}) = \sum_{l=0}^{\infty} \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} b_l(\mathbf{p}) \langle \mathbf{r} | l, \mathbf{p} \rangle, \tag{14}$$

where b_l is the anti-commuting annihilation operator. $b_l(\mathbf{p})$ obeys the non-trivial boundary condition, $b_l(\mathbf{p} + 2\pi\mathbf{N}) = e^{i\phi(p, N)} b_l(\mathbf{p})$, where $\phi(p, N) = \pi(N_x + N_y) - N_y p_x$ and

$\mathbf{N} = (N_x, N_y)$ are integers. The Fourier transform of the density $\rho(\mathbf{k}) = \int d\mathbf{r} e^{i\mathbf{k}\cdot\mathbf{r}} j^0(\mathbf{r})$ is written as

$$\rho(\mathbf{k}) = \sum_{ll'} \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) b_{l'}(\mathbf{p} - \hat{\mathbf{k}}) \langle f_l | e^{ik_x \xi + ik_y \eta} | f_{l'} \rangle e^{-\frac{i}{4\pi} \hat{k}_x (2p_y - \hat{k}_y)}, \quad (15)$$

where $\hat{\mathbf{k}} = (r_s k_x, k_y/r_s)$. Conserved charges are written in the momentum representation as

$$\begin{aligned} Q &= \sum_l \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) b_l(\mathbf{p}), \\ Q_X &= r_s \sum_l \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) (i \frac{\partial}{\partial p_x} - \frac{p_y}{2\pi}) b_l(\mathbf{p}), \\ Q_Y &= \frac{1}{r_s} \sum_l \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) i \frac{\partial}{\partial p_y} b_l(\mathbf{p}), \\ Q_J &= \sum_l \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) [l + \frac{1}{2} + \pi \{ r_s^2 (i \frac{\partial}{\partial p_x} - \frac{p_y}{2\pi})^2 + r_s^{-2} (i \frac{\partial}{\partial p_y})^2 \}] b_l(\mathbf{p}). \end{aligned} \quad (16)$$

As seen in Eq. (16), the magnetic translation in the real space is equivalent to the magnetic translation in the momentum space. The free kinetic energy is quenched in a magnetic field and the one-particle spectrum becomes flat. Therefore the free system in a magnetic field is translationally invariant in the momentum space. We show that there exists the Fermi surface in the mean field solution for the striped Hall state. The existence of the Fermi surface indicates the violation of the translational symmetry in the momentum space or in the real space.

3. Mean field solution

The mean field state is constructed as

$$|0\rangle = N_1 \prod_{\mathbf{p} \in \text{F.S.}} b_l^\dagger(\mathbf{p}) |\text{vac}\rangle, \quad (17)$$

where F.S. means Fermi sea, N_1 is a normalization constant, and $|\text{vac}\rangle$ is the vacuum state in which the $l-1$ th and lower Landau levels are fully occupied. The mean field of the two-point function for the electron field is given by

$$\langle 0 | b_l^\dagger(\mathbf{p}) b_{l'}(\mathbf{p}') | 0 \rangle = \delta_{ll'} \theta(\mu - \epsilon_{\text{HF}}(\mathbf{p})) \sum_N (2\pi)^2 \delta(\mathbf{p} - \mathbf{p}' + 2\pi \mathbf{N}) e^{i\phi(p, N)}, \quad (18)$$

where μ is a chemical potential and one-particle energy $\epsilon_{\text{HF}}(\mathbf{p})$ is determined self-consistently in the Hartree-Fock approximation[5, 6]. We assume that the magnetic field B is so strong that Landau level mixing effects can be neglected. Then we use the Hamiltonian projected to the l th Landau level

$$P_l H P_l = \omega_c N_e^*(l + \frac{1}{2}) + H^{(l)}, \quad (19)$$

$$H^{(l)} = P_l H_{\text{int}} P_l,$$

where P_l is the projection operator to the l th Landau level and N_e^* is a number of electrons occupying the l th Landau level. The free kinetic term is quenched and the total Hamiltonian is reduced to the Coulomb interaction term projected to the l th Landau level, $H^{(l)}$.

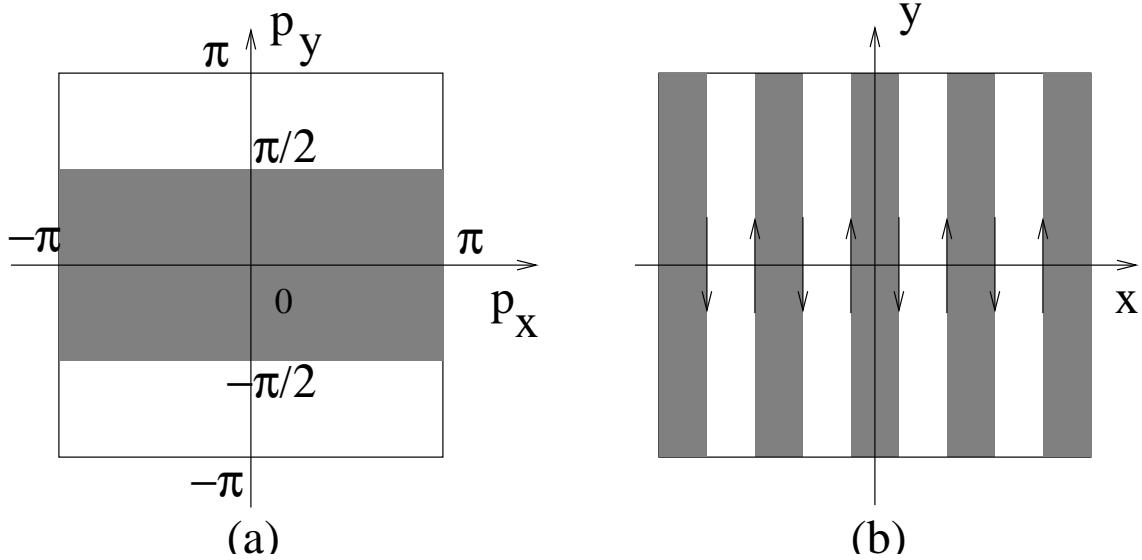


Figure 1. (a) Fermi sea for the striped Hall state at the half-filling. (b) Charge density and electric current for the striped Hall state at the half-filling.

It was shown that the mean field state with Fermi sea $|p_y| < \pi\nu_*$ satisfies the self-consistency equation at the filling factor $\nu = l + \nu_*$ ($0 < \nu_* < 1$) in the Hartree-Fock approximation [5, 6] and corresponds to the striped Hall state whose density is uniform in y direction and periodic with a period r_s in x direction[6]. The one-particle energy $\epsilon_{\text{HF}}(\mathbf{p})$ depends only on p_y in this self-consistent solution. The Fermi sea and corresponding charge density distribution in x - y space are sketched in Fig. 1. The period of stripe is r_s . The electric current flows along each stripes. Note that the true charge density and current density distribution in the mean field theory are not sharp as shown in the figure but fuzzy[6].

Hartree-Fock energy per area for the striped Hall state is calculated as

$$E_{\text{HF}} = \langle 0 | H^{(l)} | 0 \rangle / L^2 = \frac{1}{2} \int_{-\pi/2}^{\pi/2} \frac{dp_y}{2\pi} \epsilon_{\text{HF}}(p_y), \quad (20)$$

where L^2 is an area of the system. E_{HF} depends on the period of the stripe r_s and the optimal value of r_s is determined by minimizing E_{HF} . For $\nu = 2 + 1/2$ and $3 + 1/2$, optimal values are $r_s = 2.47$ and 2.88 , respectively.

The striped Hall state obtained above breaks the magnetic translation symmetry in x direction which corresponds to Q_Y . Using the commutation relation (9), we can prove the Goldstone theorem for the striped Hall state[8]. The theorem states that the NG mode appears at low energies and couples with the charge density operator. In the next section, we investigate the spectrum of the NG mode using the single mode approximation.

4. Single mode approximation

We calculate the spectrum for a neutral collective excitation at the half-filled third and fourth Landau level using the single mode approximation. The single mode approximation is successful in the FQHS because the backflow problem is absent for

the electron states projected to the Landau level[10]. Projected density operator $\rho(\mathbf{k})$ is written as $e^{-k^2/8\pi} L_l(k^2/4\pi) \rho_*(\mathbf{k})$, where

$$\rho_*(\mathbf{k}) = \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} b_l^\dagger(\mathbf{p}) b_l(\mathbf{p} - \hat{\mathbf{k}}) e^{-\frac{i}{4\pi} \hat{k}_x (2p_y - \hat{k}_y)}. \quad (21)$$

Using ρ_* , the Hamiltonian $H^{(l)}$ is written as

$$H^{(l)} = \frac{1}{2} \int \frac{d^2 k}{(2\pi)^2} \rho_*(\mathbf{k}) v_l(k) \rho_*(-\mathbf{k}), \quad (22)$$

where $v_l(k) = e^{-k^2/4\pi} (L_l(k^2/4\pi))^2 2\pi q^2/k$ for the Coulomb potential. It is well-known that the density operators projected to the Landau level are non-commutative, that is,

$$[\rho_*(\mathbf{k}), \rho_*(\mathbf{k}')] = -2i \sin\left(\frac{\mathbf{k} \times \mathbf{k}'}{4\pi}\right) \rho_*(\mathbf{k} + \mathbf{k}'). \quad (23)$$

The variational excited state is defined by $|\mathbf{k}\rangle = \rho_*(\mathbf{k})|\text{stripe}\rangle$ and the variational excitation energy $\Delta(\mathbf{k})$ is written as

$$\begin{aligned} \Delta(\mathbf{k}) &= \frac{\langle \mathbf{k} | (H_l - E_0) | \mathbf{k} \rangle}{\langle \mathbf{k} | \mathbf{k} \rangle} = \frac{f(\mathbf{k})}{s(\mathbf{k})}, \\ f(\mathbf{k}) &= \langle 0 | [\rho_*(-\mathbf{k}), [H_l, \rho_*(\mathbf{k})]] | 0 \rangle / 2N_e^*, \\ s(\mathbf{k}) &= \langle 0 | \rho_*(-\mathbf{k}) \rho_*(\mathbf{k}) | 0 \rangle / N_e^*, \end{aligned} \quad (24)$$

where E_0 is a ground state energy, N_e^* is a electron number in the l th Landau level, and $s(\mathbf{k})$ is the so-called static structure factor. To derive these expressions, we use the relation $f(-\mathbf{k}) = f(\mathbf{k})$ and $s(-\mathbf{k}) = s(\mathbf{k})$ due to π rotation symmetry. Using the commutation relation (23), $f(\mathbf{k})$ is written as

$$f(\mathbf{k}) = 2 \int \frac{d^2 k'}{(2\pi)^2} v_l(k') \sin^2\left(\frac{\mathbf{k}' \times \mathbf{k}}{4\pi}\right) \{s(\mathbf{k} + \mathbf{k}') - s(\mathbf{k}')\}, \quad (25)$$

where $v_l(k) = e^{-k^2/4\pi} (L_l(k^2/4\pi))^2 2\pi q^2/k$. Therefore the variational excitation energy is calculable if we know the static structure factor $s(\mathbf{k})$. For the mean field state (17) with $\nu_* = 1/2$, $s(\mathbf{k})$ becomes

$$\begin{aligned} s(\mathbf{k}) &= 2 \int_{\text{BZ}} \frac{d^2 p}{(2\pi)^2} \theta(\mu - \epsilon_{\text{HF}}(p_y)) \{1 - \theta(\mu - \epsilon_{\text{HF}}(p_y - \hat{k}_y))\} \\ &\quad + 2 \sum_{N_x} (2\pi)^2 \delta(\hat{k}_x + 2\pi N_x) \delta(\hat{k}_y) \left(\frac{\sin(\pi N_x/2)}{\pi N_x}\right)^2. \end{aligned} \quad (26)$$

The first term in $s(\mathbf{k})$ behaves as $|k_y|/\pi r_s$ at small k_y and periodic in k_y direction with a period $2\pi r_s$. The analytic form for $f(\mathbf{k})$ are given by using Fourier series expansion as

$$f(\mathbf{k}) = 2 \sum_{n=\text{odd}} \left\{ v_{\text{HF}}\left(\sqrt{\left(\frac{2\pi n}{r_s} + k_x\right)^2 + k_y^2}\right) - v_{\text{HF}}\left(\frac{2\pi n}{r_s}\right) \right\} \left(\frac{\sin \frac{n k_y}{2 r_s}}{\pi n}\right)^2, \quad (27)$$

$$v_{\text{HF}}(k) = v_l(k) - \int d^2 r v_l(2\pi r) e^{i\mathbf{k} \times \mathbf{r}}.$$

The numerical results for the energy spectrum Δ for $\nu = l + 1/2$, $l = 2$ and 3 are shown in Figs. 2 and 3, respectively.

As seen in these figures, the spectrum in k_y direction resembles the liquid Helium spectrum with the phonon and roton minimum. The spectrum has a multiple line node

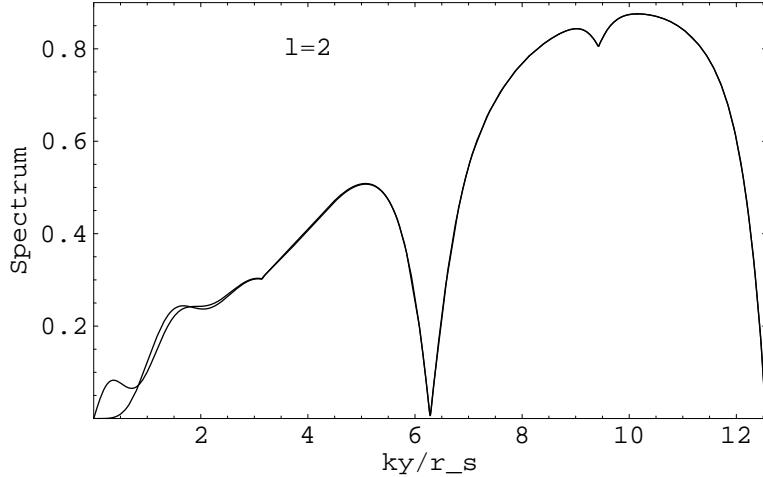


Figure 2. Energy spectrum Δ at $0 < r_s k_y < 4\pi$ for $k_x = 0$ and 1 (linear dispersion at $k_y = 0$), $\nu = 2 + 1/2$ in the single mode approximation. The unit of \mathbf{k} is a^{-1} and the unit of spectrum is q^2/a . The same unit is used in Fig. 3.

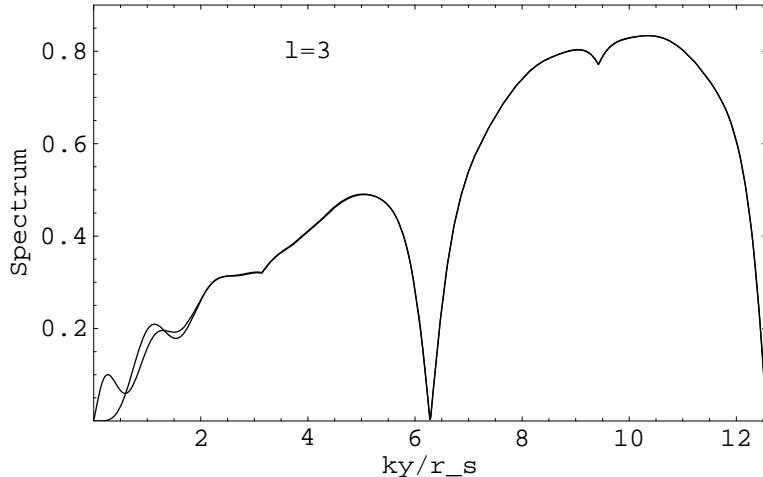


Figure 3. Energy spectrum Δ at $0 < r_s k_y < 4\pi$ for $k_x = 0$ and 1 (linear dispersion at $k_y = 0$), $\nu = 3 + 1/2$ in the single mode approximation.

at $k_y = 2\pi r_s n$ (n is integer). The comparison to particle-hole excitation energy[8] shows that the single mode approximation is good around $k_y = 2\pi r_s n$. In a usual Fermi system, the single mode approximation seems not to work well because of the particle-hole excitation near the Fermi surface at low energies. In the present case, however, the low energy excitation near the Fermi surface is suppressed by the divergence of Fermi velocity at Fermi surface[8]. Cusps also appear at $k_y = \pi r_s (2n + 1)$.

5. summary

We have studied neutral collective excitations in the striped Hall state. We have obtained the neutral excitation spectrum in the striped Hall state using the single mode approximation. The spectrum has a new rich structure. The neutral collective mode includes the NG mode due to spontaneous breaking of magnetic translation symmetry.

The spectrum is highly anisotropic, that is, the dispersion in k_x direction is similar to that of FQHS and the dispersion in k_y is similar to that of Liquid Helium. We hope these excitations will be observed in experiments for the evidence of the striped Hall state.

Acknowledgements

Two of the authors (T. A. and N. M.) thank J. L. Birman, A. Dorsey, R. M. Lewis, W. Pan, L. Tevlin, K. Yang, and J. Zhu for useful discussions. This work was partially supported by the special Grant-in-Aid for Promotion of Education and Science in Hokkaido University provided by the Ministry of Education, Science, Sport, and Culture, Japan, and by the Grant-in-Aid for Scientific Research on Priority area of Research (B) (Dynamics of Superstrings and Field Theories, Grant No. 13135201) from the Ministry of Education, Science, Sport, and Culture, Japan, and by Nukazawa Science Foundation.

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